

## Esercizi su Gram-Schmidt

Sistema n. 1

$$B' = \{(0, 1, 3, 2), (0, -2, 0, -1), (0, -3, 3, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{14}}(0, 1, 3, 2), \frac{1}{\sqrt{21}}(0, -4, 2, -1), \frac{1}{\sqrt{6}}(0, -1, -1, 2) \right\}$$

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Sistema n. 2

$$B' = \{(1, 2, -1, 1), (0, 2, 0, 0), (-2, 3, -1, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{7}}(1, 2, -1, 1), \frac{1}{\sqrt{21}}(-2, 3, 2, -2), \frac{1}{\sqrt{6}}(-2, 0, -1, 1) \right\}$$

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Sistema n. 3

$$B' = \{(-2, 0, -1, -2), (-3, 0, 0, 3), (-3, 0, 0, -1)\}$$

$$B'' = \left\{ \frac{1}{3}(-2, 0, -1, -2), \frac{1}{\sqrt{2}}(-1, 0, 0, 1), \frac{1}{3\sqrt{2}}(-1, 0, 4, -1) \right\}$$

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Sistema n. 4

$$B' = \{(-3, -2, 0, 0), (1, 1, 0, 0), (2, 1, -6, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{13}}(-3, -2, 0, 0), \frac{1}{\sqrt{13}}(-2, 3, 0, 0), \frac{1}{\sqrt{10}}(0, 0, -3, 1) \right\}$$

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Sistema n. 5

$$B' = \{(-1, 0, 0, -2), (3, 3, 1, 1), (-2, 3, 1, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{5}}(-1, 0, 0, -2), \frac{1}{\sqrt{15}}(2, 3, 1, -1), \frac{1}{\sqrt{30}}(-4, 3, 1, 2) \right\}$$

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Sistema n. 6

$$B' = \{(-2, 3, 2, -4), (2, -2, -2, 4), (-3, -2, 1, -2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{33}}(-2, 3, 2, -4), \frac{1}{\sqrt{22}}(1, 4, -1, 2), \frac{1}{\sqrt{30}}(-5, 0, -1, 2) \right\}$$

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Sistema n. 7

$$B' = \{(0, -2, 2, 0), (0, 0, -2, -1), (0, -2, -3, -3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(0, -1, 1, 0), \frac{1}{\sqrt{3}}(0, -1, -1, -1), \frac{1}{\sqrt{6}}(0, 1, 1, -2) \right\}$$

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Sistema n. 8

$$B' = \{(1, 2, 0, 0), (-1, 3, 0, -2), (-2, 0, 0, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{5}}(1, 2, 0, 0), \frac{1}{3}(-2, 1, 0, -2), \frac{1}{3\sqrt{5}}(-4, 2, 0, 5) \right\}$$

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Sistema n. 9

$$B' = \{(2, 1, 1, 3), (2, -3, -3, -1), (-1, -1, -1, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{15}}(2, 1, 1, 3), \frac{1}{\sqrt{3}}(1, -1, -1, 0), \frac{1}{\sqrt{10}}(-2, -1, -1, 2) \right\}$$

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Sistema n. 10

$$B' = \{(-3, -3, 2, 0), (0, 0, 1, 0), (1, 1, -3, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{22}}(-3, -3, 2, 0), \frac{1}{\sqrt{11}}(1, 1, 3, 0), 1(0, 0, 0, 1) \right\}$$

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Sistema n. 11

$$B' = \{(2, 0, 0, 0), (2, -3, -1, 2), (-1, 0, 3, -6)\}$$

$$B'' = \{1(1, 0, 0, 0), \frac{1}{\sqrt{14}}(0, -3, -1, 2), \frac{1}{\sqrt{70}}(0, -5, 3, -6)\}$$


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Sistema n. 12

$$B' = \{(-3, 2, -3, 0), (0, 3, 0, 0), (2, 3, 2, -3)\}$$

$$B'' = \{\frac{1}{\sqrt{22}}(-3, 2, -3, 0), \frac{1}{\sqrt{11}}(1, 3, 1, 0), 1(0, 0, 0, -1)\}$$


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Sistema n. 13

$$B' = \{(-3, -3, 1, -1), (-2, 0, 0, 0), (-3, -3, -2, 2)\}$$

$$B'' = \{\frac{1}{2\sqrt{5}}(-3, -3, 1, -1), \frac{1}{2\sqrt{55}}(-11, 9, -3, 3), \frac{1}{\sqrt{22}}(0, -2, -3, 3)\}$$


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Sistema n. 14

$$B' = \{(0, -2, -2, -1), (0, 0, -1, -3), (0, -2, 1, 3)\}$$

$$B'' = \{\frac{1}{3}(0, -2, -2, -1), \frac{1}{3\sqrt{65}}(0, 10, 1, -22), \frac{1}{\sqrt{65}}(0, -5, 6, -2)\}$$


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Sistema n. 15

$$B' = \{(0, 0, 0, -2), (3, -3, 3, 1), (-2, -3, -2, 3)\}$$

$$B'' = \{1(0, 0, 0, -1), \frac{1}{\sqrt{3}}(1, -1, 1, 0), \frac{1}{\sqrt{6}}(-1, -2, -1, 0)\}$$


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Sistema n. 16

$$B' = \{(-1, -2, 3, -1), (2, 4, 1, 2), (-1, -2, 0, 1)\}$$

$$B'' = \{\frac{1}{\sqrt{15}}(-1, -2, 3, -1), \frac{1}{\sqrt{10}}(1, 2, 2, 1), \frac{1}{\sqrt{30}}(-1, -2, 0, 5)\}$$


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Sistema n. 17

$$B' = \{(2, -3, 3, -6), (1, 3, -3, 6), (3, -2, -3, 6)\}$$

$$B'' = \{\frac{1}{\sqrt{58}}(2, -3, 3, -6), \frac{1}{\sqrt{87}}(9, 1, -1, 2), \frac{1}{\sqrt{30}}(0, -5, -1, 2)\}$$


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Sistema n. 18

$$B' = \{(-1, 1, 0, 0), (3, -3, 2, -2), (-3, 3, 1, -2)\}$$

$$B'' = \{\frac{1}{\sqrt{2}}(-1, 1, 0, 0), \frac{1}{\sqrt{2}}(0, 0, 1, -1), \frac{1}{\sqrt{2}}(0, 0, -1, -1)\}$$


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Sistema n. 19

$$B' = \{(0, 0, 1, 0), (2, -1, -3, 1), (2, -1, 0, 3)\}$$

$$B'' = \{1(0, 0, 1, 0), \frac{1}{\sqrt{6}}(2, -1, 0, 1), \frac{1}{\sqrt{30}}(-2, 1, 0, 5)\}$$


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Sistema n. 20

$$B' = \{(-3, 3, 2, 3), (-2, 2, -1, 2), (-2, 1, -3, 2)\}$$

$$B'' = \{\frac{1}{\sqrt{31}}(-3, 3, 2, 3), \frac{1}{\sqrt{93}}(-2, 2, -9, 2), \frac{1}{\sqrt{6}}(-1, -2, 0, 1)\}$$


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Sistema n. 21

$$B' = \{(2, 2, -3, -3), (1, 1, -3, -3), (2, -3, -2, -2)\}$$

$$B'' = \{\frac{1}{\sqrt{26}}(2, 2, -3, -3), \frac{1}{\sqrt{26}}(-3, -3, -2, -2), \frac{1}{\sqrt{2}}(1, -1, 0, 0)\}$$

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Sistema n. 22

$$B' = \{(1, -2, -1, 1), (-1, 1, 3, -1), (-3, -3, -1, -3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{7}}(1, -2, -1, 1), \frac{1}{\sqrt{5}}(0, -1, 2, 0), \frac{1}{\sqrt{70}}(-5, -4, -2, -5) \right\}$$

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Sistema n. 23

$$B' = \{(-2, -2, 1, 0), (-6, -2, 3, 0), (-4, -1, 2, -1)\}$$

$$B'' = \left\{ \frac{1}{3}(-2, -2, 1, 0), \frac{1}{3\sqrt{5}}(-4, 5, 2, 0), 1(0, 0, 0, -1) \right\}$$

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Sistema n. 24

$$B' = \{(0, -2, 2, 0), (0, 3, -2, 0), (-2, 3, 3, 6)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(0, -1, 1, 0), \frac{1}{\sqrt{2}}(0, 1, 1, 0), \frac{1}{\sqrt{10}}(-1, 0, 0, 3) \right\}$$

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Sistema n. 25

$$B' = \{(2, -3, 0, -1), (0, 2, 0, -1), (-2, 0, 0, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{14}}(2, -3, 0, -1), \frac{1}{3\sqrt{70}}(10, 13, 0, -19), \frac{1}{3\sqrt{5}}(5, 2, 0, 4) \right\}$$

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Sistema n. 26

$$B' = \{(3, -3, -9, 3), (1, 1, -3, -1), (-1, 2, 3, 1)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{3}}(1, -1, -3, 1), \frac{1}{2\sqrt{15}}(1, 5, -3, -5), \frac{1}{\sqrt{2}}(0, 1, 0, 1) \right\}$$

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Sistema n. 27

$$B' = \{(-1, 2, 0, -1), (1, -3, 0, 1), (-1, -2, 0, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{6}}(-1, 2, 0, -1), \frac{1}{\sqrt{3}}(-1, -1, 0, -1), \frac{1}{\sqrt{2}}(-1, 0, 0, 1) \right\}$$

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Sistema n. 28

$$B' = \{(-2, -1, -6, -3), (-3, 3, -9, -1), (-2, -1, -6, 0)\}$$

$$B'' = \left\{ \frac{1}{5\sqrt{2}}(-2, -1, -6, -3), \frac{1}{10\sqrt{7}}(-3, 21, -9, 13), \frac{1}{2\sqrt{35}}(-1, -7, -3, 9) \right\}$$

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Sistema n. 29

$$B' = \{(1, -2, 0, -1), (3, 0, 0, 3), (1, 1, 0, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{6}}(1, -2, 0, -1), \frac{1}{\sqrt{2}}(1, 0, 0, 1), \frac{1}{\sqrt{3}}(1, 1, 0, -1) \right\}$$

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Sistema n. 30

$$B' = \{(1, 0, -2, -2), (0, -2, 0, 0), (1, 0, -1, -2)\}$$

$$B'' = \left\{ \frac{1}{3}(1, 0, -2, -2), 1(0, -1, 0, 0), \frac{1}{3\sqrt{5}}(2, 0, 5, -4) \right\}$$

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Sistema n. 31

$$B' = \{(1, 2, 0, -2), (3, -3, 0, -3), (0, -1, 0, 2)\}$$

$$B'' = \left\{ \frac{1}{3}(1, 2, 0, -2), \frac{1}{3\sqrt{26}}(8, -11, 0, -7), \frac{1}{\sqrt{26}}(4, 1, 0, 3) \right\}$$

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Sistema n. 32

$$B' = \{(0, -1, 6, 2), (-2, -1, -6, -2), (3, -2, -9, -3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{41}}(0, -1, 6, 2), \frac{1}{9\sqrt{41}}(-41, -40, -6, -2), \frac{1}{9\sqrt{10}}(20, -20, -3, -1) \right\}$$


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Sistema n. 33

$$B' = \{(1, -3, -3, 1), (2, 0, -2, 0), (0, 6, 3, -2)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{5}}(1, -3, -3, 1), \frac{1}{\sqrt{30}}(4, 3, -2, -1), \frac{1}{2\sqrt{15}}(-5, 3, -5, -1) \right\}$$


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Sistema n. 34

$$B' = \{(-3, 0, 2, -3), (1, -3, -1, 1), (2, -2, 3, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{22}}(-3, 0, 2, -3), \frac{1}{10\sqrt{11}}(-1, -33, -3, -1), \frac{1}{10}(3, -1, 9, 3) \right\}$$


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Sistema n. 35

$$B' = \{(0, -3, 3, -2), (-2, -3, 3, 0), (-2, -3, 3, -1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{22}}(0, -3, 3, -2), \frac{1}{2\sqrt{55}}(-11, -3, 3, 9), \frac{1}{2\sqrt{5}}(-3, 1, -1, -3) \right\}$$


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Sistema n. 36

$$B' = \{(-1, 0, 0, -3), (-2, 0, 0, 3), (-1, -3, -3, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(-1, 0, 0, -3), \frac{1}{\sqrt{10}}(-3, 0, 0, 1), \frac{1}{\sqrt{2}}(0, -1, -1, 0) \right\}$$


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Sistema n. 37

$$B' = \{(1, -1, -1, 1), (2, 0, -2, 2), (3, 3, 3, 3)\}$$

$$B'' = \left\{ \frac{1}{2}(1, -1, -1, 1), \frac{1}{2\sqrt{3}}(1, 3, -1, 1), \frac{1}{\sqrt{6}}(1, 0, 2, 1) \right\}$$


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Sistema n. 38

$$B' = \{(0, 1, -1, 0), (6, -1, 1, -2), (-9, 2, 2, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(0, 1, -1, 0), \frac{1}{\sqrt{10}}(3, 0, 0, -1), \frac{1}{\sqrt{2}}(0, 1, 1, 0) \right\}$$


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Sistema n. 39

$$B' = \{(0, -1, 1, -1), (0, -2, 2, -3), (0, 3, -2, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{3}}(0, -1, 1, -1), \frac{1}{\sqrt{6}}(0, 1, -1, -2), \frac{1}{\sqrt{2}}(0, 1, 1, 0) \right\}$$


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Sistema n. 40

$$B' = \{(0, 2, 2, 1), (3, 2, 2, -2), (-1, -1, -1, 1)\}$$

$$B'' = \left\{ \frac{1}{3}(0, 2, 2, 1), \frac{1}{3\sqrt{17}}(9, 2, 2, -8), \frac{1}{\sqrt{34}}(4, -1, -1, 4) \right\}$$


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Sistema n. 41

$$B' = \{(1, 0, -2, -2), (1, 0, 3, -2), (-3, 0, -2, 0)\}$$

$$B'' = \left\{ \frac{1}{3}(1, 0, -2, -2), \frac{1}{3\sqrt{5}}(2, 0, 5, -4), \frac{1}{\sqrt{5}}(-2, 0, 0, -1) \right\}$$


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Sistema n. 42

$$B' = \{(-1, 1, 1, 0), (-2, -1, -1, 0), (-2, 0, 2, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{3}}(-1, 1, 1, 0), \frac{1}{\sqrt{6}}(-2, -1, -1, 0), \frac{1}{\sqrt{2}}(0, -1, 1, 0) \right\}$$

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Sistema n. 43

$$B' = \{(3, 0, 0, 9), (1, 1, 1, 3), (-3, -1, 3, -9)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(1, 0, 0, 3), \frac{1}{\sqrt{2}}(0, 1, 1, 0), \frac{1}{\sqrt{2}}(0, -1, 1, 0) \right\}$$

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Sistema n. 44

$$B' = \{(-3, -1, 1, 1), (0, 3, -3, -3), (2, -3, -2, -2)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{3}}(-3, -1, 1, 1), \frac{1}{2}(-1, 1, -1, -1), \frac{1}{\sqrt{6}}(0, -2, -1, -1) \right\}$$

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Sistema n. 45

$$B' = \{(0, 1, 0, -3), (0, -1, 0, 0), (-1, 1, 2, -2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(0, 1, 0, -3), \frac{1}{\sqrt{10}}(0, -3, 0, -1), \frac{1}{\sqrt{5}}(-1, 0, 2, 0) \right\}$$

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Sistema n. 46

$$B' = \{(-3, 0, 0, 1), (2, 1, -2, 0), (-3, 2, -4, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(-3, 0, 0, 1), \frac{1}{3\sqrt{15}}(1, 5, -10, 3), \frac{1}{3\sqrt{30}}(-5, 2, -4, -15) \right\}$$

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Sistema n. 47

$$B' = \{(0, 0, 3, 0), (2, -6, 3, 3), (3, 4, -2, -2)\}$$

$$B'' = \left\{ 1(0, 0, 1, 0), \frac{1}{7}(2, -6, 0, 3), \frac{1}{7\sqrt{5}}(15, 4, 0, -2) \right\}$$

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Sistema n. 48

$$B' = \{(-1, 1, 1, 1), (-2, -3, -3, 2), (-1, 0, 0, -2)\}$$

$$B'' = \left\{ \frac{1}{2}(-1, 1, 1, 1), \frac{1}{2}(-1, -1, -1, 1), \frac{1}{\sqrt{2}}(-1, 0, 0, -1) \right\}$$

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Sistema n. 49

$$B' = \{(-4, 1, 1, -2), (0, 2, 2, 0), (-2, 1, 2, -1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{22}}(-4, 1, 1, -2), \frac{1}{\sqrt{55}}(2, 5, 5, 1), \frac{1}{\sqrt{2}}(0, -1, 1, 0) \right\}$$

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Sistema n. 50

$$B' = \{(-1, -1, 2, -1), (0, 0, 0, -2), (-3, -3, 2, -3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{7}}(-1, -1, 2, -1), \frac{1}{\sqrt{42}}(1, 1, -2, -6), \frac{1}{\sqrt{3}}(-1, -1, -1, 0) \right\}$$

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Sistema n. 51

$$B' = \{(-2, 1, 1, -1), (0, -2, -2, 2), (-3, -1, 3, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{7}}(-2, 1, 1, -1), \frac{1}{\sqrt{21}}(-3, -2, -2, 2), \frac{1}{\sqrt{6}}(0, -1, 2, 1) \right\}$$

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Sistema n. 52

$$B' = \{(2, 1, -2, -3), (-3, -2, 3, 2), (1, 3, -1, 2)\}$$

$$B'' = \left\{ \frac{1}{3\sqrt{2}}(2, 1, -2, -3), \frac{1}{3\sqrt{34}}(-7, -8, 7, -12), \frac{1}{\sqrt{34}}(-2, 5, 2, -1) \right\}$$

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Sistema n. 53

$$B' = \{(3, -9, 3, 3), (-2, 0, 0, 2), (2, 3, -1, -1)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{3}}(1, -3, 1, 1), \frac{1}{\sqrt{2}}(-1, 0, 0, 1), \frac{1}{2\sqrt{15}}(5, 3, -1, 5) \right\}$$


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Sistema n. 54

$$B' = \{(-1, 0, 0, 3), (0, 6, 2, 0), (3, 6, 2, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(-1, 0, 0, 3), \frac{1}{\sqrt{10}}(0, 3, 1, 0), \frac{1}{\sqrt{10}}(3, 0, 0, 1) \right\}$$


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Sistema n. 55

$$B' = \{(0, 2, 0, -3), (-2, 0, -2, 1), (3, 3, 3, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{13}}(0, 2, 0, -3), \frac{1}{3\sqrt{39}}(-13, 3, -13, 2), \frac{1}{3\sqrt{6}}(1, 6, 1, 4) \right\}$$


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Sistema n. 56

$$B' = \{(-3, -1, -3, -3), (3, 1, -1, -1), (-2, -2, 3, 3)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{7}}(-3, -1, -3, -3), \frac{1}{2\sqrt{35}}(9, 3, -5, -5), \frac{1}{\sqrt{10}}(1, -3, 0, 0) \right\}$$


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Sistema n. 57

$$B' = \{(1, 3, -2, -6), (2, 1, 1, -2), (1, 3, 3, -6)\}$$

$$B'' = \left\{ \frac{1}{5\sqrt{2}}(1, 3, -2, -6), \frac{1}{5\sqrt{22}}(17, 1, 16, -2), \frac{1}{\sqrt{55}}(-5, 1, 5, -2) \right\}$$


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Sistema n. 58

$$B' = \{(-3, 2, 0, 0), (0, 0, -3, 0), (-3, -2, -1, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{13}}(-3, 2, 0, 0), 1(0, 0, -1, 0), \frac{1}{\sqrt{13}}(-2, -3, 0, 0) \right\}$$


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Sistema n. 59

$$B' = \{(2, 3, 0, 2), (-2, 0, 0, 2), (3, 3, 0, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{17}}(2, 3, 0, 2), \frac{1}{\sqrt{2}}(-1, 0, 0, 1), \frac{1}{\sqrt{34}}(-3, 4, 0, -3) \right\}$$


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Sistema n. 60

$$B' = \{(2, 2, 2, 1), (-3, -3, -3, -2), (-2, -2, -1, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{13}}(2, 2, 2, 1), \frac{1}{\sqrt{39}}(1, 1, 1, -6), \frac{1}{\sqrt{6}}(-1, -1, 2, 0) \right\}$$


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Sistema n. 61

$$B' = \{(-1, 1, 0, 0), (-3, 3, 3, 2), (-1, 1, 0, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(-1, 1, 0, 0), \frac{1}{\sqrt{13}}(0, 0, 3, 2), \frac{1}{\sqrt{13}}(0, 0, -2, 3) \right\}$$


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Sistema n. 62

$$B' = \{(-2, 1, 0, -2), (-3, -1, 0, 1), (1, 3, 0, 1)\}$$

$$B'' = \left\{ \frac{1}{3}(-2, 1, 0, -2), \frac{1}{3\sqrt{10}}(-7, -4, 0, 5), \frac{1}{3\sqrt{10}}(-1, 8, 0, 5) \right\}$$


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Sistema n. 63

$$B' = \{(-3, 1, -1, 3), (-1, 1, 0, 1), (1, 2, -1, -1)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{5}}(-3, 1, -1, 3), \frac{1}{2\sqrt{55}}(1, 13, 7, -1), \frac{1}{\sqrt{22}}(1, 2, -4, -1) \right\}$$

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Sistema n. 64

$$B' = \{(0, 0, 1, 3), (0, 0, 1, 2), (-2, -1, -3, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(0, 0, 1, 3), \frac{1}{\sqrt{10}}(0, 0, 3, -1), \frac{1}{\sqrt{5}}(-2, -1, 0, 0) \right\}$$

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Sistema n. 65

$$B' = \{(0, 1, 0, 0), (-2, 0, 0, 0), (-3, -2, 0, 3)\}$$

$$B'' = \{1(0, 1, 0, 0), 1(-1, 0, 0, 0), 1(0, 0, 0, 1)\}$$

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Sistema n. 66

$$B' = \{(-3, 0, 6, 3), (-3, 0, 6, -3), (1, -2, -2, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{6}}(-1, 0, 2, 1), \frac{1}{\sqrt{30}}(-1, 0, 2, -5), 1(0, -1, 0, 0) \right\}$$

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Sistema n. 67

$$B' = \{(2, 0, 2, 1), (-1, 0, 2, -2), (2, 0, 2, -2)\}$$

$$B'' = \left\{ \frac{1}{3}(2, 0, 2, 1), \frac{1}{3}(-1, 0, 2, -2), \frac{1}{3}(2, 0, -1, -2) \right\}$$

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Sistema n. 68

$$B' = \{(1, 2, -2, 0), (3, 0, 0, -1), (-2, 2, -2, -3)\}$$

$$B'' = \left\{ \frac{1}{3}(1, 2, -2, 0), \frac{1}{9}(8, -2, 2, -3), \frac{1}{9\sqrt{2}}(-4, 1, -1, -12) \right\}$$

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Sistema n. 69

$$B' = \{(2, -1, 0, -2), (6, -3, -2, 3), (-2, 1, 3, 0)\}$$

$$B'' = \left\{ \frac{1}{3}(2, -1, 0, -2), \frac{1}{7}(4, -2, -2, 5), \frac{1}{21\sqrt{5}}(8, -4, 45, 10) \right\}$$

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Sistema n. 70

$$B' = \{(-2, 1, 6, -3), (-3, 1, 9, -3), (2, 0, -6, 3)\}$$

$$B'' = \left\{ \frac{1}{5\sqrt{2}}(-2, 1, 6, -3), \frac{1}{5\sqrt{2}}(-1, -2, 3, 6), \frac{1}{\sqrt{10}}(0, 3, 0, 1) \right\}$$

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Sistema n. 71

$$B' = \{(-1, 1, -1, 3), (-2, 1, -2, 1), (3, -1, 3, -1)\}$$

$$B'' = \left\{ \frac{1}{2\sqrt{3}}(-1, 1, -1, 3), \frac{1}{\sqrt{42}}(-4, 1, -4, -3), \frac{1}{2\sqrt{7}}(1, 5, 1, -1) \right\}$$

---

Sistema n. 72

$$B' = \{(-2, 2, 0, 2), (1, -1, -1, 1), (-3, 3, 3, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{3}}(-1, 1, 0, 1), \frac{1}{\sqrt{33}}(2, -2, -3, 4), \frac{1}{\sqrt{22}}(1, -1, 4, 2) \right\}$$

---

Sistema n. 73

$$B' = \{(-1, 0, 0, 0), (-3, 3, 3, -2), (2, 1, 1, -1)\}$$

$$B'' = \left\{ 1(-1, 0, 0, 0), \frac{1}{\sqrt{22}}(0, 3, 3, -2), \frac{1}{\sqrt{11}}(0, -1, -1, -3) \right\}$$

---

Sistema n. 74

$$B' = \{(-1, -1, 2, -2), (1, -1, -2, -2), (1, 0, -3, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(-1, -1, 2, -2), \frac{1}{\sqrt{10}}(1, -1, -2, -2), \frac{1}{\sqrt{5}}(-2, 0, -1, 0) \right\}$$


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Sistema n. 75

$$B' = \{(-2, 2, 0, 0), (-2, 2, 6, 3), (2, 3, 0, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(-1, 1, 0, 0), \frac{1}{\sqrt{5}}(0, 0, 2, 1), \frac{1}{\sqrt{2}}(1, 1, 0, 0) \right\}$$


---

Sistema n. 76

$$B' = \{(0, 2, 0, 2), (4, 0, -2, 0), (-6, 3, 3, -3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(0, 1, 0, 1), \frac{1}{\sqrt{5}}(2, 0, -1, 0), \frac{1}{\sqrt{2}}(0, 1, 0, -1) \right\}$$


---

Sistema n. 77

$$B' = \{(-2, 2, 0, 0), (-3, 1, 0, 0), (3, -3, 6, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(-1, 1, 0, 0), \frac{1}{\sqrt{2}}(-1, -1, 0, 0), \frac{1}{\sqrt{5}}(0, 0, 2, 1) \right\}$$


---

Sistema n. 78

$$B' = \{(-1, -1, 1, 1), (3, 2, -2, -3), (1, 1, 3, -1)\}$$

$$B'' = \left\{ \frac{1}{2}(-1, -1, 1, 1), \frac{1}{2}(1, -1, 1, -1), \frac{1}{\sqrt{2}}(0, 1, 1, 0) \right\}$$


---

Sistema n. 79

$$B' = \{(2, 1, 2, -2), (2, 0, -1, -2), (0, -2, 1, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{13}}(2, 1, 2, -2), \frac{1}{9\sqrt{13}}(14, -6, -25, -14), \frac{1}{9\sqrt{2}}(1, -12, 4, -1) \right\}$$


---

Sistema n. 80

$$B' = \{(0, 2, -4, -3), (3, 2, -4, 0), (3, -2, 4, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{29}}(0, 2, -4, -3), \frac{1}{7\sqrt{29}}(29, 6, -12, 20), \frac{1}{7\sqrt{5}}(10, -3, 6, -10) \right\}$$


---

Sistema n. 81

$$B' = \{(0, 6, 3, 3), (2, -2, -3, -1), (-2, 4, 1, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{6}}(0, 2, 1, 1), \frac{1}{\sqrt{66}}(6, 2, -5, 1), \frac{1}{\sqrt{55}}(-5, 2, -5, 1) \right\}$$


---

Sistema n. 82

$$B' = \{(0, 0, -2, 0), (-3, -1, -3, 1), (0, 3, -1, -3)\}$$

$$B'' = \left\{ 1(0, 0, -1, 0), \frac{1}{\sqrt{11}}(-3, -1, 0, 1), \frac{1}{\sqrt{22}}(-2, 3, 0, -3) \right\}$$


---

Sistema n. 83

$$B' = \{(0, -2, 2, 3), (0, -3, 3, -2), (0, 3, 2, -2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{17}}(0, -2, 2, 3), \frac{1}{\sqrt{34}}(0, -3, 3, -4), \frac{1}{\sqrt{2}}(0, 1, 1, 0) \right\}$$


---

Sistema n. 84

$$B' = \{(3, -3, 3, 3), (2, -3, 3, 3), (3, -3, -3, -3)\}$$

$$B'' = \left\{ \frac{1}{2}(1, -1, 1, 1), \frac{1}{2\sqrt{3}}(-3, -1, 1, 1), \frac{1}{\sqrt{6}}(0, -2, -1, -1) \right\}$$


---



Sistema n. 85

$$B' = \{(-2, 0, 0, 1), (2, -4, 2, -1), (1, 0, 0, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{5}}(-2, 0, 0, 1), \frac{1}{\sqrt{5}}(0, -2, 1, 0), \frac{1}{\sqrt{5}}(1, 0, 0, 2) \right\}$$

---

Sistema n. 86

$$B' = \{(3, -1, -1, 2), (1, -2, -2, -1), (3, 0, 0, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{15}}(3, -1, -1, 2), \frac{1}{\sqrt{3}}(0, -1, -1, -1), \frac{1}{\sqrt{10}}(2, 1, 1, -2) \right\}$$

---

Sistema n. 87

$$B' = \{(0, 3, 1, -1), (0, 2, -1, 3), (0, -3, -3, -3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{11}}(0, 3, 1, -1), \frac{1}{5\sqrt{66}}(0, 16, -13, 35), \frac{1}{5\sqrt{6}}(0, 2, -11, -5) \right\}$$

---

Sistema n. 88

$$B' = \{(-2, 2, 3, 3), (2, -2, -2, -1), (0, 0, -1, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{26}}(-2, 2, 3, 3), \frac{1}{7\sqrt{26}}(18, -18, -1, 25), \frac{1}{7\sqrt{2}}(-3, 3, -8, 4) \right\}$$

---

Sistema n. 89

$$B' = \{(-2, 2, -1, -1), (3, 0, 0, -1), (-1, -6, 3, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(-2, 2, -1, -1), \frac{1}{\sqrt{30}}(4, 2, -1, -3), \frac{1}{\sqrt{15}}(-1, -2, 1, -3) \right\}$$

---

Sistema n. 90

$$B' = \{(0, -3, 2, -6), (2, 3, -2, 6), (-3, 0, -3, 9)\}$$

$$B'' = \left\{ \frac{1}{7}(0, -3, 2, -6), 1(1, 0, 0, 0), \frac{1}{7\sqrt{10}}(0, -20, -3, 9) \right\}$$

---

Sistema n. 91

$$B' = \{(2, 3, 2, -6), (-3, -3, -2, 6), (-2, 2, -1, -4)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{53}}(2, 3, 2, -6), \frac{1}{7\sqrt{53}}(-49, 6, 4, -12), \frac{1}{7\sqrt{5}}(0, 2, -15, -4) \right\}$$

---

Sistema n. 92

$$B' = \{(0, -2, -6, 3), (0, 3, -6, 3), (-1, 2, 2, -1)\}$$

$$B'' = \left\{ \frac{1}{7}(0, -2, -6, 3), \frac{1}{7\sqrt{5}}(0, 15, -4, 2), 1(-1, 0, 0, 0) \right\}$$

---

Sistema n. 93

$$B' = \{(-1, 0, 1, 0), (0, 1, 0, 1), (2, 3, -2, 1)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(-1, 0, 1, 0), \frac{1}{\sqrt{2}}(0, 1, 0, 1), \frac{1}{\sqrt{2}}(0, 1, 0, -1) \right\}$$

---

Sistema n. 94

$$B' = \{(0, 3, 1, 2), (3, -3, -1, 1), (1, 6, 2, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{14}}(0, 3, 1, 2), \frac{1}{2\sqrt{21}}(7, -3, -1, 5), \frac{1}{2\sqrt{15}}(5, 3, 1, -5) \right\}$$

---

Sistema n. 95

$$B' = \{(-1, -1, 2, 2), (-3, 1, 0, 0), (-1, 3, 0, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{10}}(-1, -1, 2, 2), \frac{1}{2\sqrt{15}}(-7, 3, -1, -1), \frac{1}{2\sqrt{3}}(1, 3, 1, 1) \right\}$$

---

Sistema n. 96

$$B' = \{(2, -2, -2, 0), (3, -1, -3, 0), (3, 2, 2, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{3}}(1, -1, -1, 0), \frac{1}{\sqrt{6}}(1, 2, -1, 0), \frac{1}{\sqrt{2}}(1, 0, 1, 0) \right\}$$

---

Sistema n. 97

$$B' = \{(-1, 1, -2, 1), (-2, 2, -1, 2), (-2, 2, 3, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{7}}(-1, 1, -2, 1), \frac{1}{\sqrt{21}}(-2, 2, 3, 2), \frac{1}{\sqrt{6}}(-1, 1, 0, -2) \right\}$$

---

Sistema n. 98

$$B' = \{(0, 2, -2, 0), (-1, 0, -2, 1), (0, 0, 2, 0)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{2}}(0, 1, -1, 0), \frac{1}{2}(-1, -1, -1, 1), \frac{1}{2}(-1, 1, 1, 1) \right\}$$

---

Sistema n. 99

$$B' = \{(1, 2, -2, 2), (0, -2, 2, 1), (0, -3, 3, 2)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{13}}(1, 2, -2, 2), \frac{1}{9\sqrt{13}}(6, -14, 14, 25), \frac{1}{9\sqrt{2}}(-12, 1, -1, 4) \right\}$$

---

Sistema n. 100

$$B' = \{(-1, -1, 0, -1), (0, 1, 1, 0), (3, 0, 3, 3)\}$$

$$B'' = \left\{ \frac{1}{\sqrt{3}}(-1, -1, 0, -1), \frac{1}{\sqrt{15}}(-1, 2, 3, -1), \frac{1}{\sqrt{10}}(1, -2, 2, 1) \right\}$$